REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. REPORT DATE *(DD-MM-YYYY)*	**2. REPORT TYPE**	**3. DATES COVERED** *(From - To)*
JANUARY 2012	CONFERENCE PAPER (Post Print)	JAN 2009 – DEC 2011

4. TITLE AND SUBTITLE

SPECTRUM SHAPING CHALLENGES IN DYNAMIC SPECTRUM ACCESS NETWORKS WITH TRANSMISSION HYPERSPACE

5a. CONTRACT NUMBER FA8750-10-C-0221

5b. GRANT NUMBER N/A

5c. PROGRAM ELEMENT NUMBER N/A

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5d. PROJECT NUMBER EUTH

5e. TASK NUMBER GF

5f. WORK UNIT NUMBER 10

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

ANDRO Computational Solutions, LLC
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8. PERFORMING ORGANIZATION REPORT NUMBER

N/A

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

Air Force Research Laboratory/Information Directorate
Rome Research Site/RITE
525 Brooks Road
Rome NY 13441-4505

10. SPONSOR/MONITOR'S ACRONYM(S)

N/A

11. SPONSORING/MONITORING AGENCY REPORT NUMBER

AFRL-RI-RS-TP-2012-021

12. DISTRIBUTION AVAILABILITY STATEMENT

APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED. PA Case Number: 88ABW-2011-5705
DATE CLEARED: 26 OCTOBER 2011

13. SUPPLEMENTARY NOTES

14. ABSTRACT

Our goal in this paper is to discuss several issues and challenges involved with dynamic spectrum access in cognitive radio networks (CRNs). In this context, we introduce three optimization problems for spectrum shaping in cognitive radio networks (CRNs) using orthogonal frequency division multiplexing (OFDM). These optimization problems utilize the concept of transmission hyperspace (TH). The first problem involves maximization of sum-rate of the network under primary and secondary quality-of-service (QoS) constraints utilizing frequency and space dimensions of the TH. The second problem relaxes the omnidirectional assumption and incorporates antenna directionality in the first problem. As for the third problem, time dimension of the TH is added to the first two problems and the resulting problem is formulated as a time scheduling problem. We show that all these problems are NP-hard which requires the development of efficient heuristic algorithms.

15. SUBJECT TERMS

Dynamic spectrum access, cognitive radio networks, transmission hyperspace, multi-objective optimization.

16. SECURITY CLASSIFICATION OF:			**17. LIMITATION OF ABSTRACT**	**18. NUMBER OF PAGES**	**19a. NAME OF RESPONSIBLE PERSON**
a. REPORT	**b. ABSTRACT**	**c. THIS PAGE**			MICHAEL J. MEDLEY
U	U	U	UU	5	**19b. TELEPHONE NUMBER** (*Include area code*) N/A

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std. Z39.18

Spectrum Shaping Challenges in Dynamic Spectrum Access Networks with Transmission Hyperspace

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***Abstract*—Our goal in this paper is to discuss several issues and challenges involved with dynamic spectrum access in cognitive radio networks (CRNs). In this context, we introduce three optimization problems for spectrum shaping in cognitive radio networks (CRNs) using orthogonal frequency division multiplexing (OFDM). These optimization problems utilize the concept of transmission hyperspace (TH). The first problem involves maximization of sum-rate of the network under primary and secondary quality-of-service (QoS) constraints utilizing frequency and space dimensions of the TH. The second problem relaxes the omnidirectional assumption and incorporates antenna directionality in the first problem. As for the third problem, time dimension of the TH is added to the first two problems and the resulting problem is formulated as a time scheduling problem. We show that all these problems are NP-hard which requires the development of efficient heuristic algorithms.**

I. Introduction

The emerging paradigm of Dynamic Spectrum Access (DSA) networks, also known as NeXt generation networks (xG) or Cognitive Radio Networks (CRNs), has been proposed as a solution to the problem of spectrum inefficiency especially with the rapid growth in wireless demand. Cognitive radio networking enables multiple radios with different priority levels to co-exist and co-operate in an environment without interfering with each other, so as to increase spectrum efficiency [1]. The overall objective of cognitive radio networking is to achieve maximized network efficiency without interrupting higher priority transmissions and without compromising security while jointly satisfying heterogeneous quality-of-service (QoS) requirements of multiple cognitive users. The only way to achieve such an objective is to use orthogonal transmission dimensions such as frequency, time, space, coding, and antenna directionality. We refer to the multi-dimensional transmission space in a cognitive radio network (CRN) as the *"Transmission Hyperspace (TH)"* [2], [3]. The TH concept represents a new technology for achieving enhanced dynamic spectrum access and spectrum maneuverability in the theater of operations by going beyond simply assigning or allocating frequency spectrum to networked communications systems. Achieving this objective is a very challenging task, and the solution (as well as the methodology to obtain that solution) heavily depends on the given problem specifications. In general, there is no unique (and global) method that can be applied to every cognitive radio networking optimization problem.

Although the advantages of using multiple transmit dimensions in the context of TH are obvious, the resulting optimization problems involving TH are more complex. Our goal in this paper is to illustrate these challenges by introducing three optimization problems for spectrum shaping in CRNs utilizing TH and discuss difficulties associated with solving these problems. We concentrate on a specific frequency sharing technique which is orthogonal frequency division multiplexing (OFDM) since it has been advocated as a promising candidate technology for cognitive radio networks. The first problem is to maximize the sum-rate of the network under primary and secondary QoS constraints using omnidirectional antennas. In this problem, frequency and space are the TH dimensions that are used to maximize the sum-rate of the network. In the second problem, we maximize the sum-rate but relax the constraint of omnidirectional antennas by allowing directional antennas. In this case, frequency and space are TH dimensions. However the optimization space has an additional dimension which is antenna directionality. Directional antennas are expected to improve spatial reuse hence improve the performance of the network in terms of sum-rate. As a third problem, we add time in addition to frequency and space as a third TH dimension and formulate a spectrum shaping problem to minimize total time to transmit messages of certain sizes under primary and secondary QoS constraints. We show that these optimization problems are NP-hard [4]. Therefore, one needs efficient suboptimal algorithms in order to solve them in real time.

II. Transmission Hyperspace Concept

The Transmission Hyperspace (TH) is a concept which has been proposed to address the fundamental problems of spectrum crowding by providing control of multiple orthogonal communication dimensions (time, geolocation, frequency, power, antenna beam direction, beamwidth, coding, etc.) using a system optimization approach [2], [3]. This is intended to maximize desired connectivity and throughput for intended users while concurrently denying access to unauthorized or malicious users. The TH concept is useful not only for single communication networks but also in environments where multiple communication networks co-exist with radars and multi-sensor systems. These radars and multi-sensor systems put additional constraints on the system design which complicates the shared dynamic spectrum access problem. The resulting TH based optimization problems usually boil down to spectrum shaping problems involving additional dimensions such as time and antenna parameters.

The TH paradigm regards the RF resource space as an electromagnetically occupied hypercube volume (Figure 1) existing in n dimensions (time, space, frequency, beam direction, code/modulation, etc.) [5]. Here, the space is constantly changing with “cells” of network resources that have been assigned, used, and released. The system parameters are continuously adapted based upon feedback from sensed returns. The TH approach represents a unique solution to “spectrum exploitation.” Several key capabilities are brought together under the TH concept: (i) the ability to achieve dynamic “spectrum” access that goes beyond just allocating frequencies by employing a sense and adapt approach over multiple communication dimensions to “optimize” the RF transmission plan; (ii) the use of embedded algorithms that characterize the EM environment focusing on the physical (PHY) layer; and (iii) models to study the impact on the upper layers (data, network) due to incident electromagnetic interference and disruptive jamming at the PHY layer. The frequency dimension is the fundamental parameter addressed by dynamic spectrum allocation techniques. If it is the only dimension considered then the secondary user (SU) must respect the legal rights of primary users (PUs) while also compensating for space, time and frequency variations due to multipath propagation, mobility, and location dependent shadowing. The TH concept considers additional dimensions that allows secondary users to continue transmitting even when primary users access the frequency channel by maintaining orthogonality. Consider the time dimension. Messages received in non-overlapping time intervals will not interfere with each other. The simplest approach to exploit this fact is to discretize time by division into non-overlapping intervals or time slots, and schedule one message for each time slot, ignoring the interference from messages in other time slots. Now consider directionality. The use of directional antennas drastically increases the number of messages that can be sent in the time slot. Antenna at each node can then be oriented in the direction of the node with which it is to communicate, and the signal is transmitted in a beam of limited angle originating from the sender. Consequently, such a message will not significantly interfere with the messages being received simultaneously by other nodes that do not lie close to the axis of the beam, or whose antennas are facing in substantially different directions. In our previous work [3], we have shown by simulations with a 10×10 array of nodes that the number of time slots required to transmit a message from every node to every other node can be reduced by a factor of over $23\times$ by the use of directional antennas with a beam-angle of 25 degrees, using the heuristic of allotting to each time slot those messages that interfere the least with other messages already allocated to that time slot. As the beam-angle reduces, this factor improves further. However, the beam-angle is a function of the hardware, possibly not tunable to fit a problem. In our earlier work [6], we derived explicit expressions for the successful communication probability (SCP) in a multi-hop CRN and we proved that the proposed TH concept significantly improves network utility in terms of SCP. In [6], we considered time, frequency, power, antenna directionality and beamwidth as the TH dimensions to improve SCP.

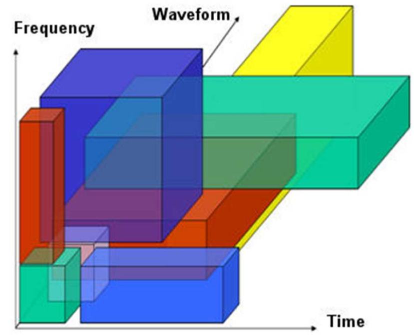

Fig. 1. Transmission Hyperspace provides multidimensional DSA.

III. System Model and Problem Statement

We consider a CRN, where the available frequency band is shared between SUs and PUs in the network with the constraint that SU transmitters do not create harmful interference at PU receivers. We assume that the shared spectrum is divided into K discrete frequency subbands and, without loss of generality, each subband has an identical bandwidth of B Hz. This set of assumptions is applicable to systems using orthogonal-frequency-division-multiplexing (OFDM) technology which has been widely advocated to be a promising candidate technology for cognitive radio networks [7]. We number each secondary and primary transmitter-receiver pair by the indices $n \in \mathcal{N} = \{SU_1, \ldots, SU_N\}$ and $m \in \mathcal{M} = \{PU_1, \ldots, PU_M\}$, respectively, and refer to them as users. Throughout the paper, the terms subband and carrier are used interchangeably. Our formulations are based on the physical model [8], which provides a realistic modeling of the physical communication environment by utilizing the path-loss model. The path loss between a transmitter-receiver pair n is given as

$$L_n(k) = \frac{G_{t,n}G_{r,n}}{d_n^{\alpha}} \left(\frac{c}{4\pi f(k)}\right)^2, \tag{1}$$

where $G_{t,n}$ and $G_{r,n}$ are the transmit and receive antenna gains of user n, respectively, c is the speed of light, $f(k)$ is the carrier frequency of subband k, d_n is the distance between transmitter n and receiver n, and α is the attenuation constant. We assume that the path-loss in the received power is the dominant loss factor, and therefore, we neglect the effects of shadowing and multi-path fading. We assume a Gaussian channel with zero mean and variance N_0, and that the received interference is treated as white noise. Under these assumptions, the achievable data rate of user n can be expressed [9] as:

$$R_n = B\sum_{k=1}^{K} \log[1 + \gamma_n(k)], \tag{2}$$

where $\log$ is defined in base 2 and $\gamma_n(k)$ is the signal-to-interference-plus-noise ratio (SINR) of user n on carrier k,

$$\gamma_n(k) \triangleq \frac{p_n(k)L_n(k)}{N_0 + \sum_{l\in\mathcal{N}\cup\mathcal{M}, l\neq n} p_l(k)L_l(k)}. \tag{3}$$

In (3), $p_n(k)$ and $p_l(k)$ represent transmit powers of the n-th user and the l-th user, respectively. The SINR condition for establishing a successful communication link n on carrier k is given by $\gamma_n(k) \geq \gamma_n^*$.

We assume a narrowband primary network, where a single channel with predetermined transmit power values is allocated to each primary user. This scenario is applicable to networks where legacy radios have the licences to operate on narrowband channels. Generalization to a wideband primary network is straightforward and does not affect the methodology. Secondary users utilize multiband techniques to access the spectrum and each secondary user has a power budget denoted by P^B.

IV. Maximization of Sum-Rate Under QoS Constraints with Omnidirectional Antennas

In this section, we consider omnidirectional antennas, i.e., $G_{t,n} = G_{r,n} = 1$, $\forall n$. We assume that each PU occupies a single subband and PUs operate on disjoint subbands. This results in the equality $M = K$. Given primary network activity and location of users, the optimization problem is to maximize the sum-rate (or achievable capacity) of the secondary network. The optimization variables are power levels allocated to each secondary user over each shared frequency subband.

Define $\mathbf{p}_n \triangleq [p_n(1), \ldots, p_n(K)]^T$ as the power allocation vector where each element represents the power level allocated to user n over each subband. User n is said to be inactive over frequency band k if $p_n(k) = 0$. A user is said to be active if it is transmitting on at least one subband. Let $\mathcal{F}_n$ denote the set of frequency channels with nonzero power allocations for session n, which implies $|\mathcal{F}_n| \leq K$. The notation $|\cdot|$ represents the cardinality of a set. In this case, our setting requires that $|\mathcal{F}_m| = 1$ for all $m \in \mathcal{P}$, and $\mathcal{F}_i \cap \mathcal{F}_j = \emptyset$ if $i \neq j$ and $i, j \in \mathcal{P}$. An SU is allowed to transmit on a carrier, if and only if it does not violate any SINR or power budget constraints.

The sum-rate maximization problem, P1, is formulated as follows:

$$\begin{aligned}
\text{Find} \quad & \mathbf{p}_n, \quad \forall n \in \mathcal{SU} \\
\text{Maximize} \quad & \sum_{n=1}^{N} R_n(\mathbf{p}_n) && (4)\\
\text{Subject to} \quad & \gamma_m(k) \geq \gamma_m^*, \quad \forall m \in \mathcal{M}, \quad \forall k \in \mathcal{F}_m, && (5)\\
& \gamma_n(k) \geq \mathbf{I}(p_n(k))\gamma_n^*, \quad \forall n \in \mathcal{N}, \quad \forall k, && (6)\\
& p_n(k) \geq 0, \quad \forall n \in \mathcal{N}, \quad \forall k, && (7)\\
& \sum_{k=1}^{K} p_n(k) \leq P^B, \quad \forall n \in \mathcal{N}. && (8)
\end{aligned}$$

In P1, $\mathbf{I}(\cdot)$ is the indicator function for the set of positive real numbers. Inequality (5) represents a set of $K(= M)$ SINR constraints for the primary users. The inequality (6) represents a set of $N \times K$ SINR constraints for each of N SUs over each of K frequency subbands. The SU n will transmit at a frequency k, if and only if the SINR of that link is greater than or equal to the threshold SINR. If the link is not active over that subband, i.e., if $p_n(k) = 0$, the SINR constraint is automatically satisfied.

Proposition 1: The sum-rate maximization problem with QoS constraints P1 is NP-hard.

Proposition 1 can be proven by noticing that P1 is a generalization of the sum-rate maximization problem without the QoS constraints which was shown to be NP-hard [10].

V. Maximization of Sum-Rate with Under QoS Constraints with Directional Antennas

In the previous section, we assumed that all users transmit with omnidirectional antennas. In this section, we relax that constraint and consider directional antennas at transmitters. In this setting, users transmit using directional antennas and receive using omnidirectional antennas. This can be realized by transmit beamforming techniques. Transmit directionality is expected to increase network throughput and decrease delay by providing enhanced spatial reuse and increased range [11]. For the antenna pattern, we consider the so-called keyhole model [6] which is simple but still reflects the main characteristics of a realistic antenna pattern as shown in Figure 2. Using this keyhole pattern, we are able to model both the enhanced power gain in the main-lobe region and the reduced power gain in the side-lobe region compared to an isotropic antenna pattern.

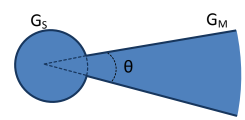

Fig. 2. Keyhole Antenna Pattern.

The derivation of different antenna gain values that provide a fair comparison with the case using omnidirectional antennas is provided in [6]. We omit the details due to space limitations and briefly explain those calculations here for completeness. Let θ denote the main lobe beamwidth in radians. We define an additional main lobe beamwidth denoted by θ' also in radians, where $\theta + \theta' \leq 2\pi$. Then we define the normalized beamwidths as $\alpha := \theta/2\pi$ and $\eta := \theta'/2\pi$. After some manipulations, the transmit antenna gains are calculated as

$$G_m = \frac{1}{\alpha + \eta}, \; G_s = \frac{\eta}{(1-\alpha)(\alpha+\eta)}. \tag{9}$$

Note that the purpose of introducing θ' is that the main lobe and the side lobe gains can be controlled by adjusting θ'. The interested reader is referred to [6] for further details. In this problem formulation, everything stays the same as in the previous formulation (in Section IV) except different transmit gains G_M and G_S in the main lobe and the side lobe, respectively. Given the locations of users, the new path loss for user n is now a function of the variable transmit antenna direction ϕ_n where. A communication link setup with transmit directionality is illustrated in Figure 3.

Note that the interference term in (3) which is the summation of the power terms from all other transmitters is also a function of other transmitters' transmit directions. We now augment the power vector defined in Section IV

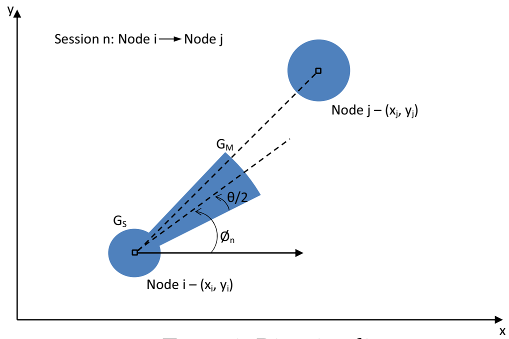

Fig. 3. Transmit Directionality

with the transmit direction angle and define the new vector $\mathbf{q}_n \triangleq [p_n(1), \ldots, p_n(N_f), \phi_n]^T$ for session n. The sum-rate maximization problem $P2$ is defined as:

$$\begin{aligned}
&\text{Find} && \mathbf{q}_n, \quad \forall n \in \mathcal{SU} \\
&\text{Maximize} && \sum_{n=1}^{N} R_n(\mathbf{q}_n) && (10)\\
&\text{Subject to} && \gamma_m(k) \geq \gamma_m^*, \quad \forall m \in \mathcal{M}, \quad \forall k \in \mathcal{F}_m, && (11)\\
& && \gamma_n(k) \geq \mathbf{I}(p_n(k))\gamma_n^*, \quad \forall n \in \mathcal{N}, \quad \forall k, && (12)\\
& && p_n(k) \geq 0, \quad \forall n \in \mathcal{N}, \quad \forall k, && (13)\\
& && \sum_{k=1}^{K} p_n(k) \leq P^B, \quad \forall n \in \mathcal{N}. && (14)
\end{aligned}$$

The problem $P2$ is also NP-hard and it is even more complex than the previous problem because of the additional optimization dimensions ϕ_n, $n = 1, \ldots, N$.

VI. Minimization of Total Time to Transmit

The formulations of the previous problems are based on a single time slot. In other words, they are one-shot waveform design problems. In this section, we add time dimensionality of the TH into the optimization problem and combine the waveform shaping problem with the time scheduling problem. The time minimization problem $P3$ is formulated as follows:

$$\begin{aligned}
&\text{Find} && \mathbf{p}_n^t, \quad t = 1, \ldots, T^c, \quad \forall n \in \mathcal{SU} \\
&\text{Minimize} && T^c && (15)\\
&\text{Subject to} && \sum_{t=1}^{T^c} R_n(\mathbf{p}_n^t) = L_n/T_s, \; \forall n \in \mathcal{S}, && (16)\\
& && \gamma_m^t(k) \geq \gamma_m^*, \quad \forall m \in \mathcal{M}, \; \forall k \in \mathcal{F}_m, \; \forall t && (17)\\
& && \gamma_n^t(k) \geq \mathbf{I}(p_n^t(k))\gamma_n^*, \quad \forall n \in \mathcal{N}, \; \forall k, \; \forall t && (18)\\
& && p_n^t(k) \geq 0, \quad \forall n \in \mathcal{N}, \quad \forall k, \; \forall t && (19)\\
& && \sum_{k=1}^{K} p_n^t(k) \leq P^B, \quad \forall n \in \mathcal{N}, \; \forall t. && (20)
\end{aligned}$$

In $P3$, the time dependent power allocation vector for user n is defined as $\mathbf{p}_n^t \triangleq [p_n^t(1), \ldots, p_n^t(K)]^T$ as the time dependent power allocation vector where each element represents the power level allocated to each subband at time slot t. Other additional variables in $P3$ are defined as follows. T_s denotes the length of a time slot in seconds. Each secondary user n has a corresponding message of size L_n bits. Using the Shannon capacity R in (2) as a surrogate for data rate, the number of bits that can be sent in a time slot can be calculated as RT_s. T^c is the total time required to complete all the secondary sessions. We should note that $P3$ can also be formulated using directional antennas by replacing $\mathbf{p}_n^t$ by $\mathbf{q}_n^t$. $P3$ is also an NP-hard optimization problem.

VII. Discussion

We have introduced three practical optimization problems for waveform shaping in OFDM CRNs. These problems utilize multiple orthogonal transmit dimensions which provide the basis for our TH concept explained in II. Although it is intuitive that adding more dimensions to the TH improves spectrum efficiency, the optimization problems become harder with increasing number of dimensions. We have shown that the optimization problems formulated in Sections IV-VI are NP-hard. Therefore, we need efficient heuristic algorithms to solve these problems. Our future goal is to devise time efficient optimization algorithms for these problems and explore additional TH dimensions.

Acknowledgment

This material is based on research sponsored in part by the Air Force Research Laboratory, under agreement number FA8750-10-C-0221. The views and conclusions contained herein are those of the authors and should not be interpreted as necessarily representing the official policies or endorsements, either expressed or implied, of the Air Force Research Laboratory or the U.S. Government.

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